Period & frequency of Simple Harmonic Motion

Defining equation for SHM: $a = -\omega^2 x$

where ω is used to find the frequency or period of the oscillation:

$$\omega = \frac{2\pi}{T} = 2\pi f$$

units: s^{-1}

$$\left(\text{recall } T = \frac{1}{f} \text{ or } f = \frac{1}{T}\right)$$

Example - the period of a pendulum:

Recall: $a = -\frac{g}{\ell}x$ and $a = -\omega^2 x$

$$\omega^2 = \frac{g}{\ell}$$

$$\omega = \sqrt{\frac{g}{\ell}}$$

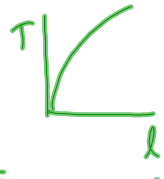
$$\frac{2\pi}{T} = \sqrt{\frac{g}{\ell}}$$

$$\frac{T}{2\pi} = \sqrt{\frac{\ell}{g}}$$

- note that the amplitude doesn't affect period
- mass doesn't affect the period.

$$\boxed{T = 2\pi\sqrt{\frac{\ell}{g}}}$$

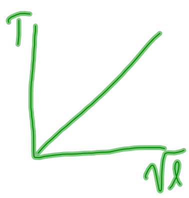
T vs ℓ

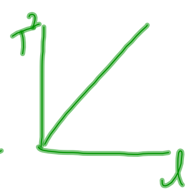


$$T^2 = \frac{4\pi^2}{g}\ell$$

a graph of T^2 vs ℓ will linear with a slope $\frac{4\pi^2}{g}$ and a y-int. of zero

$T \propto \sqrt{\ell}$ $T^2 \propto \ell$





The slope could be used to find a value for g:

$$\text{Slope} = \frac{4\pi^2}{g}$$

Data Booklet

$\omega = \dfrac{2\pi}{T}$ → $\omega = 2\pi f$

$\underbrace{x = x_0 \sin\omega t \; ; \quad v = v_0 \cos\omega t \; ;}_{\text{projection on the vertical axis}}$

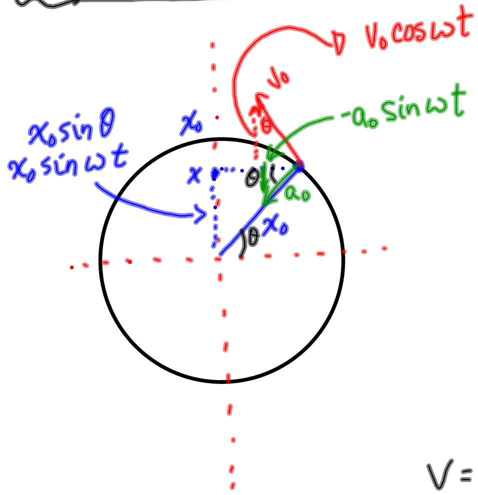
$\boxed{\begin{array}{l} x = x_0\cos\omega t \\ v = -v_0 \sin\omega t \\ v = -x_0\omega\sin\omega t \end{array}}$

projection on horizontal axis

$a = -\omega^2 x$ ← MUST KNOW!

$a = -\omega^2 x_0 \cos\omega t$

Projected circular motion - with projection on the vertical axis.



The solution to the defining equation $a = -\omega^2 x$ and the corresponding values of v and a become: (projecting on the vertical axis)

$x = x_0 \sin\omega t$

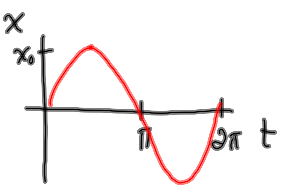
$v = v_0\cos\omega t$ or $v = x_0\omega\cos\omega t$

$a = -a_0 \sin\omega t$ or $a = -x_0\omega^2 \sin\omega t$

$a = -\omega^2 x$

If $x = 0$ at $t = 0$

$x = x_0 \sin\omega t$

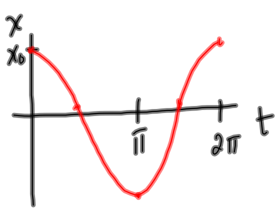


$v = v_0 \cos\omega t$

$(a = -a_0 \sin\omega t)$

If $x = x_0$ at $t = 0$

$x = x_0 \cos\omega t$



$v = -v_0 \sin\omega t$

$(a = -a_0\cos\omega t)$

Relationship between displacement x and velocity v

$$v = v_0 \cos \omega t$$

$$v^2 = v_0^2 \cos^2 \omega t \qquad 1 = \sin^2\theta + \cos^2\theta$$

$$v^2 = v_0^2 (1 - \sin^2 \omega t) \qquad (v_0 = x_0 \omega)$$

$$v^2 = x_0^2 \omega^2 (1 - \sin^2 \omega t)$$

$$v^2 = \omega^2 (x_0^2 - x_0^2 \sin^2 \omega t) \qquad (x = x_0 \sin \omega t)$$

$$v^2 = \omega^2 (x_0^2 - x^2)$$

$$v = \pm \omega \sqrt{(x_0^2 - x^2)}$$

← don't need to know t in order to find v.

⊖ left / ⊕ right

RADIANS!

EXAMPLE:

A pendulum has a period of 1.2 s and an amplitude of 0.10 m. Calculate the displacement, velocity, and acceleration of the pendulum bob 0.70 s after it is released. $t = 0,\ x = x_0$

$T = 1.2\,s$

$x_0 = 0.10\,m$

$t = 0.70\,s$

$x = ?$

$v = ?$

$a = ?$

Since the pendulum is released from its maximum displacement and that is when the timing starts, we use: $x = x_0 \cos \omega t$ etc

$v = -v_0 \sin \omega t$

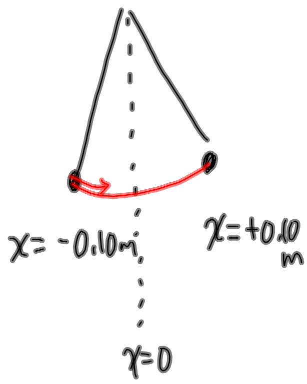
$a = -a_0 \cos \omega t$ or $a = -\omega^2 x$ easier!

First find ω:

$$\omega = \frac{2\pi}{T}$$

$$\omega = \frac{2\pi}{1.2\,s}$$

$$\boxed{\omega \doteq 5.2\,s^{-1}}$$



$$x = x_0 \cos \omega t$$

$$x = (0.10m) \cos\left((5.2s^{-1})(0.7s)\right)$$

$$\boxed{x = -0.087\,m}$$

$$v = -v_0 \sin \omega t$$

$$v = -x_0 \omega \sin \omega t$$

$$v = -(0.10m)(5.2s^{-1}) \sin\left((5.2s^{-1})(0.7s)\right)$$

$$\boxed{v = +0.25\,m/s}$$

going to the right.

OR $v = \pm \omega \sqrt{x_0^2 - x^2}$

you will have to choose which is approp

$$a = -\omega^2 x$$

$$a = -(5.2s^{-1})^2(-0.087m)$$

$$\boxed{a = +2.4\,ms^{-2}}$$

acceleration is to the right (Fnet is to the right (bob is to the left ...